LESSON 6.1 Algebraic Expressions

Engage

ESSENTIAL QUESTION
How do you add, subtract, factor, and multiply algebraic expressions? Sample answer: You can use the properties of addition and the Distributive Property to add and subtract algebraic expressions by combining like terms. You can use the Distributive Property to multiply and factor algebraic expressions.

Motivate the Lesson
Ask: Can you think of some quantities that vary and some quantities that stay the same?

Explore

Connect to Daily Life
Discuss with students how temperature and prices are examples of quantities that vary, but the length of a day and the year someone was born are examples of quantities that stay the same. Brainstorm other examples of quantities that vary and quantities that stay the same.

Explain

EXAMPLE 1
Connect Multiple Representations
Make sure students understand that an algebraic expression is another way of representing the information from a verbal expression.

Questioning Strategies
• Why is the Commutative Property used to simplify the expression? It allows the order of the addends to be switched so that the like terms are together.
• What property is used to combine like terms? Distributive Property

YOUR TURN
Avoid Common Errors
In Your Turn 3, students may forget to distribute the negative sign to the second term within the parentheses. Remind students to distribute the negative sign to each term.

EXAMPLE 2
Connect Multiple Representations
Make sure students understand that a percent can be changed to a decimal or a fraction.

Questioning Strategies
• How could you write the expression representing what the band gets to keep using a fraction instead of a decimal? \( \frac{3}{4} \times (16.60a + 12.20c) \).
• What expression could you write to represent how much the band does not get to keep? Sample answer: \( \frac{1}{4} \times (16.60a + 12.20c) \).
**LESSON 6.1 Algebraic Expressions**

**ESSENTIAL QUESTION**

How do you add, subtract, factor, and multiply algebraic expressions?

**Adding and Subtracting Expressions**

You can use the properties of addition along with the Distributive Property to add and subtract algebraic expressions.

**EXAMPLE 1**

Jill and Kyle get paid per project. Jill is paid a project fee of $25 plus $10 per hour. Kyle is paid a project fee of $18 plus $14 per hour. Write an expression to represent how much a company will pay to hire both to work the same number of hours on a project.

**STEP 1**

Write expressions for how much the company will pay each person. Let \( h \) represent the number of hours they will work on the project.

- Jill: \( 25 + 10h \)
- Kyle: \( 18 + 14h \)

**STEP 2**

Add the expressions to represent the amount the company will pay to hire both.

\[
25 + 10h + 18 + 14h
\]

Combine like terms.

\[
25 + 18 + 10h + 14h
\]

Use the Commutative Property.

\[
43 + 24h
\]

Combine like terms.

- The company will pay \( 43 + 24h \) dollars to hire both Jill and Kyle.

**Reflect**

1. **Critical Thinking**

   What can you read directly from the expression \( 43 + 24h \) that you cannot read directly from the equivalent expression \( 25 + 10h + 18 + 14h \)?

   The total amount the company pays in fees and per hour.

**YOUR TURN**

Simplify each expression.

2. \[
\left(3x + \frac{1}{2}\right) + \left(7x - 4\frac{1}{2}\right)
\]

3. \[
-0.25x - 3 - (1.5x + 1.4)
\]

4. \[
63k + 42m
\]

5. \[
0.2(3x - 15c)
\]

6. \[
\frac{1}{3}(6e + 9f - 21g)
\]

7. \[
7.20 ÷ (4 + 1), 4x = 16
\]

**PROFESSIONAL DEVELOPMENT**

**Integrate Mathematical Practices MP.4.1**

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to model with mathematics. Defining variables links the symbols to their real-world meanings. Substituting values for the variables allows students to interpret the results in the context of the situation. Students use the sum of expressions to model the situation, to solve the problem, and to answer the question in context.

**Math Background**

An algebraic expression is a mathematical statement constructed from at least one variable and possibly one or more operation symbols and one or more numbers. The table shows examples and nonexamples of algebraic expressions.

### Algebraic Expressions

<table>
<thead>
<tr>
<th>Examples</th>
<th>Nonexamples</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x, 3y - 5, -2xy, z^2 + 1 )</td>
<td>( 7, 20 ÷ (4 + 1), 4x = 16 )</td>
</tr>
</tbody>
</table>

Note that an algebraic expression must not contain an equal sign. A mathematical statement that contains an equal sign, such as \( 4x = 16 \), is an equation.

**Using the Distributive Property**

You can use the Distributive Property to remove the parentheses from an algebraic expression like \( 3x + 5 \). Sometimes this is called “simplifying” or “expanding” the expression. Multiply the quantity in front of parentheses by each term within parentheses: \( 3(x + 5) = 3x + 3 \cdot 5 = 3x + 15 \).
YOUR TURN

Avoid Common Errors
When multiplying a constant by a sum or difference, students may only multiply the first term by the constant. Have them draw arrows from the constant to each term in the parentheses to help them remember to distribute fully.

EXPLORE ACTIVITY

Engage with the Whiteboard
After finding one rectangular arrangement in Part B, have students try to think of another possible rectangular arrangement of the tiles. Have a volunteer draw it on the board and use it to write another factored expression: 2(2x + 4).

Questioning Strategies Mathematical Practices
- What are the dimensions of each tile? The x-tile is x units long and 1 unit wide. The 1-tiles are 1 unit long and 1 unit wide.
- What do the dimensions of the rectangle represent? the factors

YOUR TURN

Focus on Modeling Mathematical Practices
Have students use algebra tiles to model each exercise. Point out that as the coefficients and constants get larger, modeling becomes more unwieldy.

Elaborate

Talk About It
Summarize the Lesson
Have students complete a graphic organizer, such as the one shown here, to describe how to add, subtract, multiply, and factor expressions.

GUIDED PRACTICE

Engage with the Whiteboard
Have students volunteer to fill in the blanks in Guided Practice 1–4 and draw algebra tiles to represent the factoring for Guided Practice 5.

Avoid Common Errors
Exercise 6 Remind students that there are several ways to factor the expression. The correct answer is found by placing the greatest possible factor outside the parentheses.
**Factoring Expressions**

A factor is a number that is multiplied by another number to get a product. To factor is to write a number or an algebraic expression as a product.

**Factor 4x + 8.**

**A** Model the expression with algebra tiles.

- Use 4 positive x tiles and 8 + 1-tiles.

**B** Arrange the tiles to form a rectangle. The total area represents 4x + 8.

Sample answer:

**C** Since the length multiplied by the width equals the area, the length and the width of the rectangle are the factors of 4x + 8. Find the length and width.

The length is \( \frac{1}{2} \) x tile and \( \frac{3}{2} \) + 1-tiles, or 4.

The width is 4 + 1-tiles, or 4.

**D** Use the expressions for the length and width of the rectangle to write the area of the rectangle, 4x + 8, in factored form. \( 4(x + 2) \)

**Reflect**

8. **Communicate Mathematical Ideas** How could you use the Distributive Property to check your factoring? Multiply the factors. The result should be the original expression.

**YOUR TURN**

Factor each expression.

9. \( 2x + 2 \)

10. \( 3x + 9 \)

11. \( 5x + 15 \)

12. \( 4x + 16 \)

2(\( x + 1 \))

3(\( x + 3 \))

5(\( x + 3 \))

4(\( x + 4 \))

**Guided Practice**

1. The manager of a summer camp has 14 baseballs and 23 tennis balls. The manager buys some boxes of baseballs with 12 baseballs to a box and an equal number of boxes of tennis balls with 16 tennis balls to a box. Write an expression to represent the total number of baseballs and tennis balls.

   (Example 1)

   **STEP 1** Write expressions for the total number of baseballs and tennis balls.

   - baseballs: \( 14 + 12n \)
   - tennis balls: \( 23 + (16)n \)

   **STEP 2** Find an expression for the total number of balls.

   - Combine the two expressions.
   - Use the Commutative Property.
   - Combine like terms.

   \( 14 + 23 + 12n + 16n \)

   \( = 37 + 28n \)

   - So, the total number of baseballs and tennis balls is \( 37 + 28n \).

   Use the Distributive Property to expand each expression. (Example 2)

   3. \( 0.5(12m - 22n) \)

   \( 0.5(12m - 22n) = 0.5(12m) - 0.5(22n) \)

   \( = 6m - 11n \)

   **Factor each expression.** (Example 3)

   5. \( 2x + 12 \)

   \( = 2(x + 6) \)

   6. \( 12x + 24 \)

   \( = 12(x + 2) \)

   7. \( 7x + 35 \)

   \( = 7(x + 5) \)

**DIFFERENTIATE INSTRUCTION**

**Modeling**

Provide students with algebra tiles. Have students model expressions such as the following.

- \( (2x + 8) + (4x - 3) \)
- \( (7x + 5) - (3x + 1) \)
- \( (-4x + 2) - (x + 6) \)

Students should combine tiles of the same type or remove zero pairs to create simplified models of the expressions. Have the students write the simplified expressions.

**Visual Cues**

When multiplying a constant by a sum or difference, have students first draw arrows from the constant to each term inside the parentheses. The arrows remind students to also distribute the multiplication to the other terms. For example.

\( 0.4(9x + 5y) = 0.4(9x) + 0.4(5y) \)

**Additional Resources**

**Differentiated Instruction** includes:

- Reading Strategies
- Success for English Learners (ELL)
- Reteach
- Challenge (PRE-AP)
6.1 LESSON QUIZ

A company sets up a food booth and a game booth at the county fair. The fee for the food booth is $100 plus $5 per day. The fee for the game booth is $50 plus $7 per day.

1. Write an expression for how much both booths will cost for the same number of days.
2. How much does the company pay for both booths for 5 days?

Simplify each expression.
3. $(-0.75x + 6) - (2.5x - 1.9)$
4. $8(5x - 3y)$

Factor each expression.
5. $4x + 20$
6. $6x + 54$

Lesson Quiz available online

Answers
1. $100 + 5d + 50 + 7d$, or $150 + 12d$
2. $210$
3. $-3.25x + 7.9$
4. $40x - 24y$
5. $4(x + 5)$
6. $6(x + 9)$

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GUIDED AND INDEPENDENT PRACTICE

<table>
<thead>
<tr>
<th>Concepts &amp; Skills</th>
<th>Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1 Adding and Subtracting Expressions</td>
<td>Exercises 1, 2, 9, 10, 14, 15</td>
</tr>
<tr>
<td>Example 2 Using the Distributive Property</td>
<td>Exercises 3, 4, 13</td>
</tr>
<tr>
<td>Explore Activity Factoring Expressions</td>
<td>Exercises 5–9, 11, 12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Depth of Knowledge (D.O.K.)</th>
<th>Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10</td>
<td>2 Skills/Concepts</td>
<td>MP.4.1 Modeling</td>
</tr>
<tr>
<td>11–12</td>
<td>2 Skills/Concepts</td>
<td>MP.5.1 Using tools</td>
</tr>
<tr>
<td>13</td>
<td>3 Strategic Thinking</td>
<td>MP.3.1 Logic</td>
</tr>
<tr>
<td>14–15</td>
<td>2 Skills/Concepts</td>
<td>MP.2.1 Reasoning</td>
</tr>
<tr>
<td>16</td>
<td>2 Skills/Concepts</td>
<td>MP.1.1 Problem Solving</td>
</tr>
<tr>
<td>17</td>
<td>2 Skills/Concepts</td>
<td>MP.2.1 Reasoning</td>
</tr>
<tr>
<td>18</td>
<td>3 Strategic Thinking</td>
<td>MP.7.1 Using Structure</td>
</tr>
<tr>
<td>19</td>
<td>3 Strategic Thinking</td>
<td>MP.3.1 Logic</td>
</tr>
</tbody>
</table>

**H.O.T.**

**Additional Resources**

Differentiated Instruction includes:
- Leveled Practice Worksheets

**CLUSTER CONNECTION**

Exercises 9 and 10 combine concepts from the Florida cluster “Use properties of operations to generate equivalent expressions.”
**6.1 Independent Practice**

Write and simplify an expression for each situation.

9. A company rents out 15 food booths and 20 game booths at the county fair. The fee for a food booth is $100 plus $5 per day. The fee for a game booth is $50 plus $7 per day. The fair lasts for $d$ days, and all the booths are rented for the entire time. Write and simplify an expression for the amount in dollars that the company is paid. $15(100 + 5d) + 20(50 + 7d) = 2,500 + 215d$

10. A rug maker is using a pattern that is a rectangle with a length of 96 inches and a width of 60 inches. The rug maker wants to increase each dimension by a different amount. Let $l$ and $w$ be the increases in inches of the length and width. Write and simplify an expression for the perimeter of the new pattern. $2(96 + l) + 2(60 + w) = 312 + 2l + 2w$

In 11–12, identify the two factors that were multiplied together to form the array of tiles. Then identify the product of the two factors.

11. $\begin{array}{ccc} & & \Box \\ & & \Box \\ & & \Box \end{array}$ 3 and $x + 2$; $3x + 6$

12. $\begin{array}{ccc} & & \Box \\ & & \Box \\ & & \Box \end{array}$ 4 and $2x - 1$; $8x - 4$

13. Explain how the figure illustrates that $6(9) = 6(5) + 6(4)$. The area is the product of the length and width ($6 \times 9$). It is also the sum of the areas of the rectangles separated by the dashed line ($6 \times 5$ and $6 \times 4$). So, $6(9) = 6(5) + 6(4)$.

In 14–15, the perimeter of the figure is given. Find the length of the indicated side.

14. $x + 3$  $2x + 4$  $3x - 7$  $2x + 6$

Perimeter $= 6x$

15. $x + 3$  $2x + 4$  $3x - 7$  $2x + 6$

Perimeter $= 10x + 6$

**16. Persevere in Problem Solving** The figures show the dimensions of a tennis court and a basketball court given in terms of the width $x$ in feet of the tennis court.

![Tennis Court](image)

![Basketball Court](image)

a. Write an expression for the perimeter of each court. $T: 6x + 12$, $B: 7x + 36$

b. Write an expression that describes how much greater the perimeter of the basketball court is than the perimeter of the tennis court. $x + 24$

c. Suppose the tennis court is 36 feet wide. Find all dimensions of the two courts. $T: 36$ ft by 78 ft, B: 50 ft by 94 ft

**H.O.T.**

17. **Draw Conclusions** Use the figure to find the product $(x + 3)(x + 2)$. (Hint: Find the area of each small square or rectangle, then add.) $(x + 3)(x + 2) = x^2 + 5x + 6$

18. **Communicate Mathematical Ideas** Desmond claims that the product shown at the right illustrates the Distributive Property. Do you agree? Explain why or why not. Agree. To find $58 \times 23$, let $23 = 3 + 20$. Then find the product $58(3 + 20)$. First step: $58(3) = 174$. Second step: $58(20) = 1,160$. Third step: $174 + 1,160 = 1,334$. So, $58(23) = 58(3) + 58(20)$

19. **Justify Reasoning** Describe two different ways that you could find the product $8 \times 997$ using mental math. Find the product and explain why your methods work.

(1) Think of 997 as 1,000 $-$ 3. So, $8 \times 997 = 8(1,000) - 3$.

By the Distributive Property, $8(1,000 - 3) = 8,000 - 24 = 7,976$.

(2) Think of 997 as 900 $+$ 97. By the Distributive Property, $8(900 + 90 + 7) = 7,200 + 720 + 56 = 7,976$.

**EXTEND THE MATH**

**Challenge**

A rectangle with a length of $x + 5$ has a perimeter of $4x + 14$.

1. Write the expression for the width of the rectangle in terms of $x$.

2. Suppose the perimeter of the rectangle is 42 inches. What are the length and width of the rectangle?

3. Write the expression for the area of the rectangle in terms of $x$.

4. What is the area of the rectangle when $x = 7$?

1. $x + 2$

2. length is 12 in., width is 9 in.

3. $x^2 + 7x + 10$

4. 108 in$^2$

**Activity available online** my.hrw.com